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A new failure criterion for GFRP composite materials subjected to in-phase and out-of-phase biaxial fatigue loading under different stress ratios

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Abstract— this studying the fatigue behavior represents one of the most important targets for any new material before being used. This is because; fatigue behavior cannot be predicted and the fatigue failure cannot be expected as in the case of static yielding. The experimental fatigue tests were conducted on thin-walled tubular specimens woven-roving glass fiber reinforced polyester (GFRP), with 0^o, 45^o and 90^o phase shift between bending and torsional moments for two fiber orientations, ($[\pm45]_2$ and $[0,90]_2$), at different negative stress ratios, R = - 1, -0.75, -0.5, -0.25, 0. A new term was introduced to the Tsai-Hahn criterion to govern the fatigue behavior of the tested specimens, considering the interact effect between the local stresses and taking into account the variations of the values of (R) , the ratio of (A/B), and phase angle (z) . Good agreement between predictions and experimental results was obtained for both in-phase and out-of-phase loadings. The new fatigue model is verified by applying it to different experimental data provided by other researchers. The obtained results by the new fatigue model are in good agreements with the experimental data of E-glass/epoxy of unidirectional plies.

Index Terms— Fatigue, Glass fiber, Polyester, Combined moments, Fluctuating stresses, Failure criteria, composite materials, Out-of-phase.

1. INTRODUCTION

Choosing the suitable failure criterion represents the main target for many researchers working with materials, and it represents the first step for new materials before being used in the field. Considering composite materials, specifically, makes it more challenging, because of their very special behavior and characteristics. Besides, it must be noted that, the suitability of a certain criterion differs greatly according to the tested material, and its stress state. Based on constant amplitude fatigue, the theories available for predicting fatigue life can be divided into three categories: Fatigue life models, Phenomenological models, and Progressive damage models. Fatigue life models are based on S-N curves or Goodman–type diagrams. They predict number of cycles to failure without taking into account damage accumulation and at which fatigue failure occurs under fixed load conditions, the present work is considered as one of the studies of that category [1].

Hashin and Rotem [2] proposed first fatigue failure criteria and developed a fiber failure mode and a matrix-failure mode, expressed as $\sigma_A = \sigma_A^u, \left(\frac{\sigma_T}{\sigma_T^u}\right) + \left(\frac{\tau}{\tau^u}\right) = 1,$

where $\sigma_A and \sigma_T$ are the stresses along the fibers and transverse to the fibers, τ is the shear stress and $\sigma_A^u, \sigma_T^u and \tau^u$ are the ultimate tensile, ultimate transverse tensile and ultimate shear stress respectively. The criteria was expressed in three S-N curves which were determined experimentally on testing off-axis unidirectional specimens under uniaxial load as the ultimate strengths are function of fatigue stress level, stress ratio and number of cycles. This criteria is valid only for laminates with unidirectional piles which exhibit two failure modes during fatigue.

Reifsnider and Gao [3] proposed a failure criteria which is similar to Hashin and Rotem, the only difference is that they considered average stresses σ_{ij}^m and σ_{ij}^f in matrix and fibers. Their criterion is based on average stress formulation of composite materials derived from Mori-Tanaka method (a method to calculate the average stress fields in homogeneities and their surrounding matrix). The failure functions for the two failure mechanisms are $\sigma_{11}^f = X^f, \left(\frac{\sigma_{22}^m}{X^m}\right)^2 + \left(\frac{\sigma_{12}}{S^m}\right)^2 = 1$

where $X^f$and X^m are fatigue failure functions under tensile loading for fiber and unreinforced matrix materials and S^m is the fatigue failure function of the unreinforced matrix under shear loading. The failure functions depend on the stress ratio R, the number of cycles N, the frequency f, and S-N curves which are determined experimentally.

Philippidis,V.[4] proposed a multiaxial fatigue criteria similar to Tsai-Wu quadratic failure criteria for static loading and expressed as $F_{ij}\sigma_i\sigma_j + F_i\sigma_j - 1 \le 0$; i, j = 1, 2, 6

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where F_{ij} and F_i are functions of number of cycles N , the stress ratio R and the frequency of loading v . The tensor components F_{ij} and F_i are calculated using the S-N curves which were determined assuming $X_t = X_c$ and $Y_t = Y_c$, where X_t, X_y, Y_t, Y_c are static failure stresses of the material. They have used laminate properties to predict laminate behavior instead of lamina properties as the S-N curves for the laminate account for various damage types occurring in different types of composite materials. Their criteria predicted acceptable fatigue failure under multiaxial loading but for each laminate stacking sequence new series of experiments should be conducted. The well-known failure criteria are listed for anisotropic materials for plane stress conditions in Ref. [6].

Atcholi K. E. et al. [5] concluded that, using Hill's function could give good results if the fatigue cycle is symmetric (zero mean stress) and therefore, the smaller difference between tensile and compressive strength has to be accounted for. This is true for unidirectional glass fibre reinforced epoxy. For other materials with more complex structures, complicated functions, as Tsai-Wu criterion may be required. The paper is organized as follows: Section 2 discusses the experimental work. Section 3 introduces the test results. Section 4 is applicability of failure criteria. Section 5 gives conclusions.

2. Experimental Work

Woven-roving E-glass fibers and polyester resin, with trade name of “siropol 8330”, were used to produce the used specimens. Table 1 shows the properties of the tested materials [ASTM. D2150-63]. This resin was prepromoted with Cobalt Naphthenate (6% solution), as an accelerator in percentage of 0.2 % by volume, and Methyl Ethyl Ketone (M.E.K.) peroxide as a catalyst in a percentage of 2% by volume, depending on room temperature. The volume fraction (Vf), in the present work, ranges from 55% to 65% was used; because this range has proved its suitability to ensure specimens with good strength, good adhesion between fibers and matrix, and acceptable mechanical properties.

TABLE 1
Properties Of Used Materials

Woven–roving E-glass fibers		Polyester	
Property	Value	Property	Value
Density	2551 kg /m3	Density	1161.3 kg / m3
Modulus of elasticity	E = 76 GPa	Modulus of elasticity	E = 3.5 GPa
Poisson’s ratio	υ = 0.37	Poisson’s ratio	υ = 0.25
Tensile strength	3.45 GPa	Gel time at 25o C	20 min.
Average mass / area	600 gm / m2	Viscosity	0.45 Pa.s
Average thickness	0.69 mm	Percentage of Styrene	40 %
Weave	plain	Trade name	Siropol 8340

Thin-walled tubular specimens were used for the experimental work to ensure having a plane uniform stress. Fig.1 shows the dimensions of the used specimens. These dimensions are similar to those used by pervious investigators [6,7]. To avoid the failure of some specimens at the end of the gauge length, beneath the grippers, two wooden plugs were inserted into the specimens from both ends. And an elastic sleeve was shrinked on the outer diameter at both ends. A strain controlled testing machine, previously designed by Abouelwafa M.N. et al [8]. It is a constant speed machine of 525 rpm (8.75 Hz); and capable of performing pure torsion, pure bending, or combined torsion and bending (in-phase or out-of-phase) fatigue tests. Fig. 2. shows a general layout for the machine, the loading systems (torsion and bending) are independent, and have the facility to apply different mean stresses.

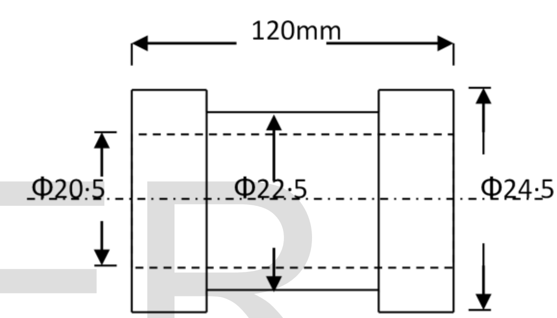

Fig.1 Dimensions of used specimens

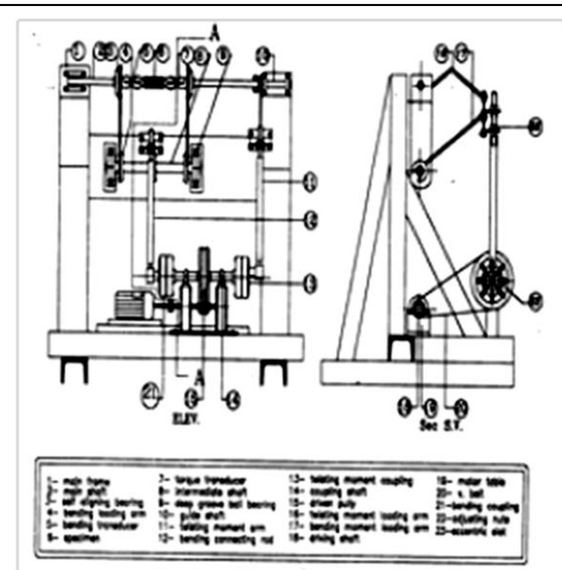
Fig.2 General layout of the testing machine

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3. Test Results

Specimens were fatigue tested under ambient conditions and constant frequency of 8.75 Hz. For each orientation $[0,90]_2$ and $[\pm 45]_2$, the data points were used to plot the corresponding S-N curves on a semi-log scale, being fitted using the power low: Max. Stress = a N^b
Table2 lists the two constants a and b for the corresponding S-N curve for $[0,90]_2$, tested in-phase, with different stress ratios (R = -1, -0.75, -0.5, -0.25, 0) at the ratio of the flexural stress (A) to the torsional shear stress (B), A/B= 2, while Table3 is for $[\pm 45]_2$ at A/B=1. In order to study the effect of mean stress in the presence of a phase difference between the bending and the torsional moments (Z), tests were performed on both fiber orientations, $[0,90]_2$ and $[\pm 45]_2$, at four different stress ratios (R = -1, -0.75, -0.5, -0.25). Table 4 lists the two constants a and b for the corresponding S-N curves for $[0,90]_2$ at all stress ratios with A/B=C/D=2 and Z=45° and 90°, while Table5 is for $[\pm 45]_2$ at A/B=1.

TABLE 2
Fatigue Constants (a) and (b) of $[0,90]_2$ Specimens with A/B=2 & Z=0°

Stress ratio (R)	a. (MPa)	b.	Correlation factor
-1	103.93	-0.1286	0.9748
-0.75	113.92	-0.1271	0.9967
-0.5	126.11	-0.1266	0.994
-0.25	141.85	-0.1267	0.9965
0	170.17	-0.1265	0.9837

TABLE 3
Fatigue Constants (a) and (b) of $[\pm 45]_2$ Specimens with A/B=1 & Z=0°

Stress ratio (R)	a. (MPa)	b.	Correlation factor
-1	103.93	-0.1286	0.9748
-0.75	113.92	-0.1271	0.9967
-0.5	126.11	-0.1266	0.994
-0.25	141.85	-0.1267	0.9965
0	170.17	-0.1265	0.9837

TABLE 4
Fatigue Constants (a) and (b) of $[0,90]_2$ Specimens with A/B=2

R	Z=90°		Z=45°	
	a.	b.	a. (MPa)	b.
-1	89.51	-0.112	95.83	-0.119
-0.75	93.47	-0.106	100.41	-0.114
-0.5	104.04	-0.108	113.18	-0.115
-0.25	109.52	-0.103	121.04	-0.113

TABLE 5
Fatigue Constants (a) and (b) of $[\pm 45]_2$ Specimens with A/B=1

R	Z=90°		Z=45°	
	a.	b.	a. (MPa)	b.
-1	77.94	-0.12	82.7	-0.127
-0.75	81.69	-0.112	87.2	-0.119
-0.5	85.72	-0.108	-	-
-0.25	95.76	-0.104	-	-

4. Applicability of failure criteria

All failure criteria have their right hand side to be unity and the left hand side contains the local stress components divided by their corresponding strength. To evaluate the validity of the failure criteria, the right hand side of the failure criteria was considered as a relative damage (R.D). The relation between the relative damage (R.D) with number of cycles to failure (N) is plotted for different failure criteria. In these curves as much as relative damage (R.D) is close to unity, this means that the criterion is suitable. If it is less than unity, then the criterion is predicting a specimen life more than the actual life of the experimental results.
The pervious works for failure criterion used in composite materials showed that the Tsai-Hahn is the more suitable failure criteria for the tension-compression stress state but it may be modified to make the R.D. of this theory near, as possible as we can, to unity.

$$\left(\frac{1}{F_{1t}}-\frac{1}{F_{1c}}\right)\sigma_1+\left(\frac{1}{F_{2t}}-\frac{1}{F_{2c}}\right)\sigma_2+\frac{\sigma_1^2}{F_{1t}F_{1c}}+\frac{\sigma_2^2}{F_{2t}F_{2c}}+2H_{12}\sigma_1\sigma_2+\left(\frac{\sigma_6}{F_6}\right)^2=1 \quad (1)$$

$$H_{12}=-0.5\sqrt{\frac{1}{F_{1t}F_{1c}F_{2t}F_{2c}}}$$

A new procedure for adapting this criterion was proposed to best fit the tested case. This procedure was based mainly on introducing a new term to increase the correlation between the experimental data and the theoretical equations.

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The Main Principals for Selecting the New Term are:

- It must depend on the local stress and strength components and not the global ones.
- It should reflect the effect of interact between the local stresses σ_1 and σ_6
- It must take into consideration the effect of variation of stress ratio (R), the ratio of q (A/B), and phase angle (z). It must be dimensionless.

The previous principals had led us to suggest introducing the term $H_{16}\left(\frac{\sigma_1\sigma_6}{F_1F_6}\right)$.

Where,

H16 is the normal and shear stresses interaction factor H16 = f(R, Z, q, F1, F6)

The suggested criterion will be as follows:

$$\left(\frac{1}{F_{1t}}-\frac{1}{F_{1c}}\right)\sigma_1+\left(\frac{1}{F_{2t}}-\frac{1}{F_{2c}}\right)\sigma_2+\frac{\sigma_1^2}{F_{1t}F_{1c}}+\frac{\sigma_2^2}{F_{2t}F_{2c}}+2H_{12}\sigma_1\sigma_2+\left(\frac{\sigma_6}{F_6}\right)^2+H_{16}\sigma_1\sigma_6=1$$

The comparison between the values of R.D. of Tsai-Hahn criterion and the suggested one for both fiber orientations $[0, 90]_2$ and $[\pm 45]_2$ specimens are plotted against the number of cycles to failure as shown in Fig.3 to Fig.6. These figures show that the suggested term has shown excellent results modifying Tsai-Hahn criterion, Equation (1), where, the values of R.D. are around the theoretical value, unity. And the difference may be referred to scatter in experimental data.

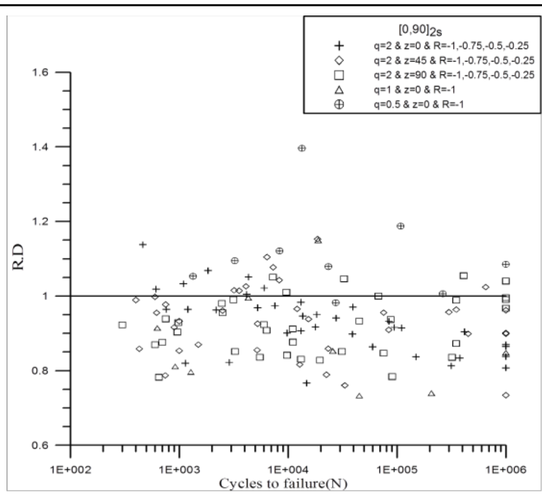

Fig.3 R.D of Tsai-Hahn theory for the $[0, 90]_2$ specimens

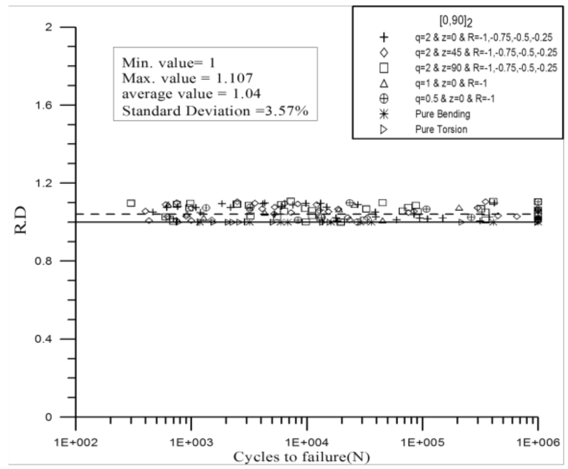

Fig. 4 R.D of the suggested failure criterion for the $[0, 90]_2$ specimens

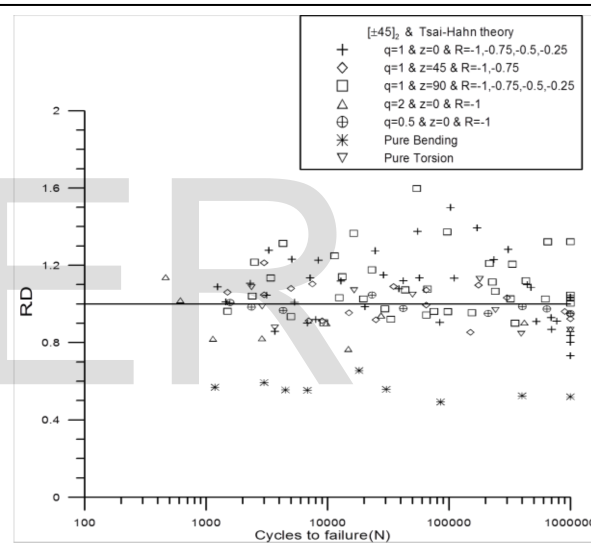

Fig.5 R.D of Tsai-Hahn theory for the $[\pm 45]_2$ specimens

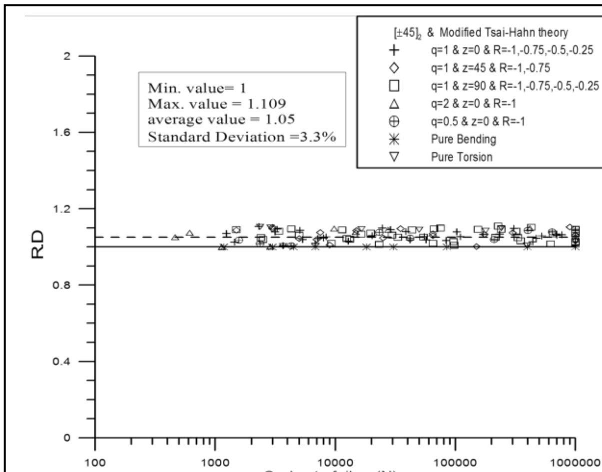

Fig.6 R.D of the suggested failure criterion for the $[\pm 45]_2$ specimens

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In order to confirm the modified Tsai-Hahn equation, data was obtained from El-Midany A. A. [6] his specimens had nearly the same specifications as those used in this work, being woven-roving GFRP tubular specimens with (V_f) ranging from 50% to 64% and two fiber orientations $[0,90]_2$ and $[\pm45]_2$. He found that the Tsai-Hahn failure criterion with it's from as in Equation (1) was the most suitable theory to predict the fatigue life of his experimental data but with relatively high R.D values and high standard deviation as shown in Fig. 7. for the $[0, 90]_2$ specimens with average value of R.D equal to 1.19 and standard deviation of 13.6 %, while, with average value of R.D. equal to 1.18 and standard deviation of 16.1 % for the $[\pm45]_2$ specimens, as shown in Fig. 8. Fig,9. and Fig.10. represent the R.D. for the same set of data adapted from El-Midany A. A. [6] but using the suggested failure criterion, for both $[0,90]_2$ and $[\pm45]_2$ fiber orientations, respectively. These Figures show that an improving in the values of R.D and standard deviation for both fiber orientations when using the modified Tsai-Hahn equation. The average value of R.D for the $[0, 90]_2$ specimens has been improved from 1.19 to 1.03 and standard deviation from 13.6 % to 2.7% and the average value of R.D for the $[\pm45]_2$ specimens has been improved from 1.18 to 1.06 and standard deviation from 16.1 % to 4.4%.

In order to check the validity of suggested failure criterion, presented in this work, for using in a wide range of different materials under different types of loading conditions, data was obtained from Atcholi KE. et al. [5]. They used a unidirectional glass/epoxy composite material with fiber volume fraction (V_f) of 64%. They tested their specimens under completely reversed combined bending and torsional stresses in-phase with zero off-axis angles. The local stresses for these specimens are $(\sigma_1 = \sigma_x, \sigma_2 = 0, \sigma_6 = \tau_{xy})$. Fig.11 represents the R.D using Equation (1), while Fig. 12 represents the R.D using the suggested failure criterion. These Figures show an improving in the values of R.D and standard deviation when using the modified Tsai-Hahn equation. The average value of R.D has been improved from 0.66 to 1.09 and standard deviation from 16.2 % to 9.1%. Using the same procedure, Fig.13 and Fig.14 show the change of R.D form 0.853 to 1.07 and standard deviation from 7.52 % to 3.44% using the suggested failure criterion for the data adapted from Kawakami H. et al. [9]. For the data adapted from Amijima S. et al. [10]. Fig. 15 and Fig.16 show the change of R.D form 0.86 to 1.11 and standard deviation from 6.4 % to 5.8% using the suggested failure criterion.

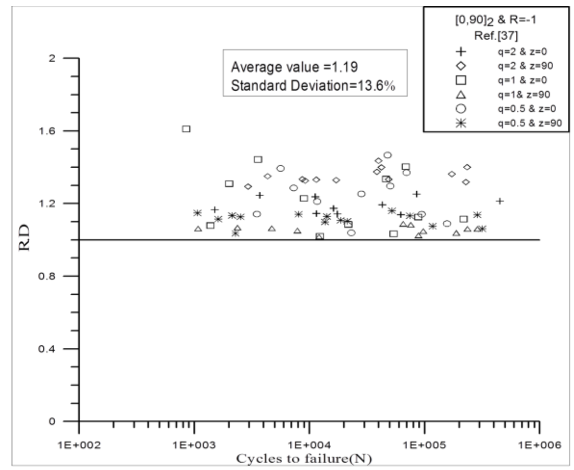

Fig.7 R.D of Tsai-Hahn theory for the $[0,90]_2$ specimens, (Data adapted from El-Midany A. A. [6])

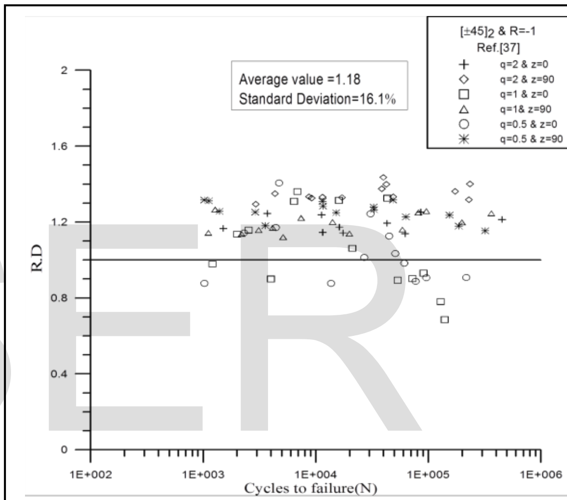

Fig.8 R.D of Tsai-Hahn theory for the $[\pm45]_2$ specimens, (Data adapted from El-Midany A. A. [6])

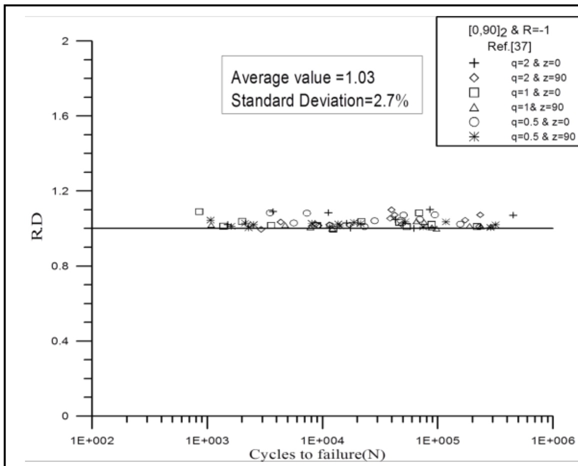

Fig.9 R.D of suggested failure criterion for the $[0,90]_2$ specimens, (Data adapted from El-Midany A. A. [6])

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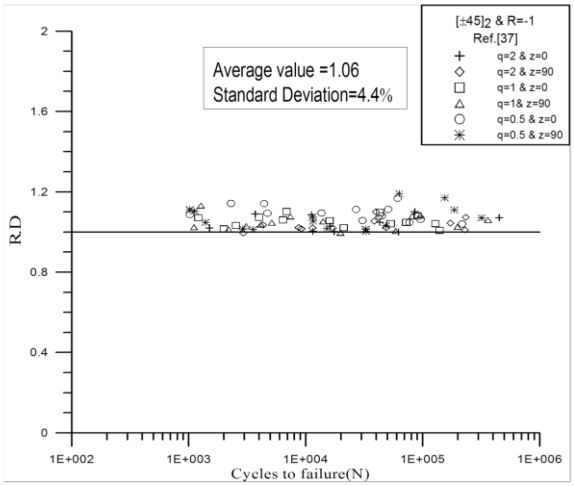

Fig.10 R.D of suggested failure criterion for the $[\pm45]_2$ specimens, (Data adapted from El-Midany A. A. [6])

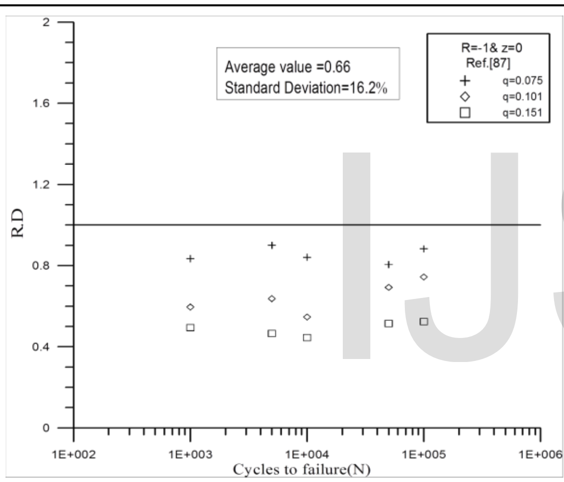

Fig.11 R.D of Tsai-Hahn criteria for data adapted from Atcholi KE. et al. [5].

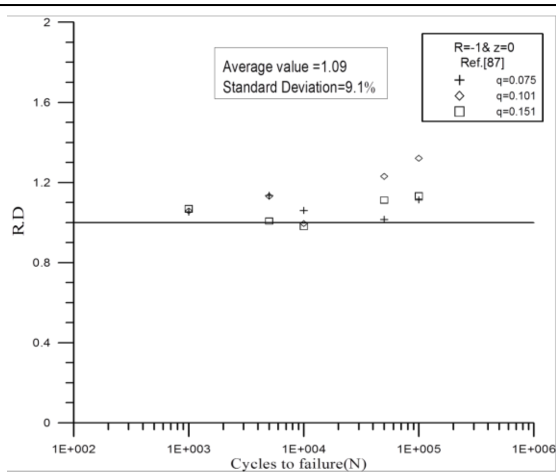

Fig.12 R.D of suggested failure criterion for data adapted from Atcholi KE. et al. [5].

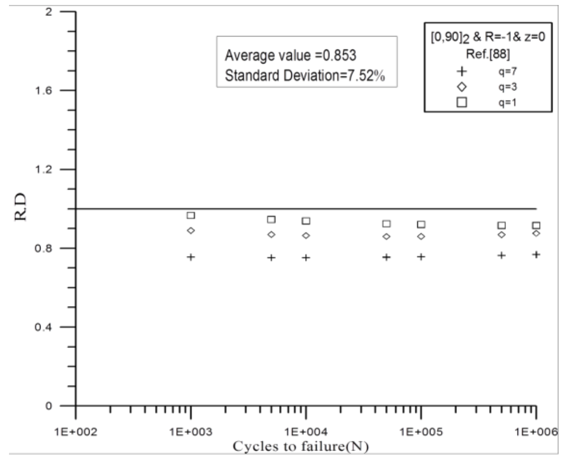

Fig.13 R.D of Tsai-Hahn criteria for data adapted from Kawakami H. et al. [9].

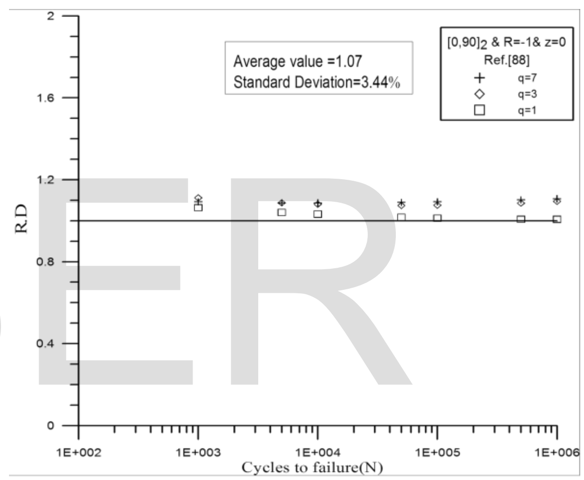

Fig.14 R.D of suggested failure criterion for data adapted from Kawakami H. et al. [9].

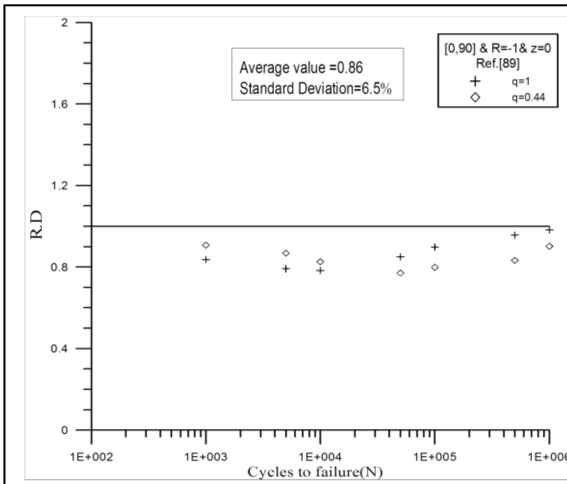

Fig.15 R.D of Tsai-Hahn criteria for data adapted from Amijima S. et al. [10].

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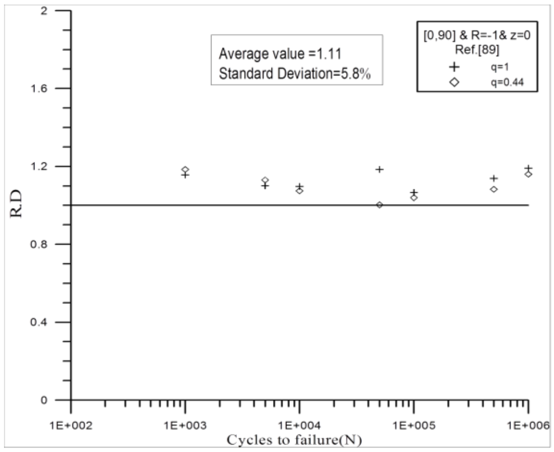

Fig.16 R.D of suggested failure criterion for data adapted from Amijima S. et al. [10].

5. Conclusion

A new fatigue failure criterion is introduced in this research for GFRP composites. In the new fatigue model all stress components and their interactions are responsible for fatigue failure. The new criterion shows excellent agreement between predictions and experimental data of the fatigue tests conducted on thin-walled tubular specimens woven-roving glass fiber reinforced polyester (GFRP), with 0^o, 45^o and 90^o phase shift between bending and torsional moments for two fiber orientations, ($[\pm45]_2$ and $[0,90]_2$), at different negative stress ratios, R = - 1, - 0.75, -0.5, -0.25, 0. Also, the new fatigue model is in good agreements with the experimental data reported in the literature of E-glass/epoxy of unidirectional plies.

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